1 # Modifications to WRF's dynamical core to improve the

2 # treatment of moisture for large-eddy simulations

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16 September 28, 2015

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20 **Abstract**

21 Yamaguchi and Feingold (2012) note that the cloud fields in their large-eddy simulations

22 (LESs) of marine stratocumulus using the Weather Research and Forecasting (WRF) model

23 exhibit a strong sensitivity to time stepping choices. In this study, we reproduce and

24 analyze this sensitivity issue using two stratocumulus cases, one marine and one

25 continental. Results show that (1) the sensitivity is associated with spurious motions near

26 the moisture jump between the boundary layer and the free atmosphere, and (2) these

27 spurious motions appear to arise from neglecting small variations in water vapor mixing

28 ratio (q_v) in the pressure gradient calculation in the acoustic sub-stepping portion of the

29 integration procedure. We show that this issue is remedied in the WRF dynamical core by

30 replacing the prognostic equation for the potential temperature θ with one for the moist

31 potential temperature $\theta_m = \theta(1+1.61q_v)$, which allows consistent treatment of moisture in

32 the calculation of pressure during the acoustic sub-steps. With this modification, the

33 spurious motions and the sensitivity to the time stepping settings (i.e., the dynamic time

34 step length and number of acoustic sub-steps) are eliminated in both of the example

35 stratocumulus cases. This modification improves the applicability of WRF for LES

36 applications, and possibly other models using similar dynamical core formulations, and

37 also permits the use of longer time steps than in the original code.

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1. Introduction

The Weather Research and Forecasting (WRF) model [Skamarock et al., 2008] is a mesoscale numerical weather model designed for atmospheric research and operational forecasting. In recent years, WRF has gained popularity as a large-eddy simulation (LES) model due to its built-in nesting capability, its open and modularized structure that facilitates the implementation of new physical parameterizations, and its fully compressible nonhydrostatic dynamic core [e.g., Moeng et al. 2007; Wang et al. 2009; Wang and Feingold 2009a, b; Yamaguchi and Feingold 2012; Blossey et al. 2013; Yamaguchi et al. 2013; Xiao et al. 2014; Endo et al. 2015].

Yamaguchi and Feingold [2012] (hereafter YF12) performed LESs using WRF v3.3.1 for two stratocumulus cases developed by the boundary layer cloud working group of the Global Energy and Water Cycle Experiment (GEWEX) Cloud System Study (GCSS; currently, Global Atmospheric System Studies (GASS)) panel. The cases are based on the first and second research flights (RF01 and RF02) of the Second Dynamics and Chemistry of Marine Stratocumulus field study (DYCOMS-II) field campaign [Stevens et al. 2003; Stevens et al. 2005; Ackerman et al. 2009]. YF12 showed that the simulated stratocumulus cloud fraction and liquid water path (LWP) depend sensitively on the choice of the dynamic integration time step (Δt) and the number of acoustic sub-steps (N_{aco}) used in each time step (see their Figure 5); this dependence is hereafter referred to as the convergence issue. For example, they found that in the DYCOMS-II RF01 case the simulated LWP shows evidence of convergence only when Δt is reduced to 0.1 s and N_{aco} is increased to 10 or 12. These time-stepping choices lead to acoustic Courant numbers far smaller than those required by the Courant-Friedrichs-Lewy (CFL) criterion. Based on this criterion, YF12 should have been able to use Δt=0.2 s and $N_{aco} \geq 4$ for their 35 m horizontal grid spacing. They concluded by advising "a convergence test be performed every time the model configuration is changed," which can be a tedious and computationally expensive requirement for LES applications.

In this study, we identify the cause of this convergence issue and present a solution for WRF. In Section 2, the issue is demonstrated for one of the aforementioned marine stratocumulus cases (the DYCOMS-II RF02 case) and a continental stratocumulus case. Evidence is provided showing that this convergence issue arises from spurious motions

70   occurring at levels with large vertical moisture gradients, e.g., at the trade wind inversion

71   in the DYCOMS-II cases. Section 3 demonstrates that these spurious motions are due to the

72   omission of the effect of small moisture perturbations on the pressure perturbations

73   during the acoustic sub-step integration as formulated in the current WRF model. A

74   modification to WRF's dynamical core is presented in Section 4, which allows consistent

75   treatment of moisture in the calculation of pressure during the acoustic sub-steps, along

76   with demonstration that this modification eliminates the spurious motions and resolves

77   the convergence issue. Finally, a summary of the findings is presented in Section 5.

78   **2. The convergence issue and spurious motions near the inversion**

79   **2.1 DYCOMS-II RF02 case**

80   We first explore the convergence issue using the DYCOMS-II RF02 marine

81   stratocumulus case, one of the cases examined in YF12, which represents nocturnal

82   stratocumulus clouds in a quasi-steady marine boundary layer capped by a strong

83   inversion. The case derives from the RF02 flight during the DYCOMS-II field campaign

84   conducted off the coast of California during July 2001 [Stevens et al. 2003; Ackerman et al.

85   2009].

86   For this case we use WRF 3.6.1 and configure the model following the original case

87   specification by Ackerman et al. [2009]. The model domain is 6.4 km × 6.4 km × 1.5 km

88   with 50 m horizontal grid spacing and 96 vertical layers. The vertical grid is stretched such

89   that the grid spacing near the surface and inversion, with a minimum of 5 m, is smaller than

90   the grid spacing in the middle of the boundary layer or in the free atmosphere. We use the

91   simple longwave radiation scheme following Stevens et al. [2005] and the Lin microphysics

92   scheme [Lin et al. 1983] modified to include cloud water sedimentation following

93   Ackerman et al. [2009]. Surface fluxes and large-scale subsidence are also prescribed

94   following Ackerman et al. [2009]. Figure 1 shows the initial profiles for the DYCOMS-II

95   RF02 case.

96   The results from simulations with Δt = 0.5 s and various N_{aco} settings are given in

97   Figure 2, which shows time series of LWP, cloud-base and cloud-top heights, and total

98   cloud fraction. The ensemble mean and spread from the LES simulations reported in

99   Ackerman et al. [2009] are also shown for comparison. The results for simulations with

100 N_{aco} = 10 and 12 are in close agreement for all four cloud variables, indicating a converged

101 solution, which lies within the ensemble range from Ackerman et al. [2009]; however, the

102 simulation with N_{aco} = 6 is markedly different, particularly in terms of LWP and total cloud

103 fraction, indicating non-convergence. The cloud fraction and LWP for N_{aco} = 8 are slightly

104 lower than those for N_{aco} = 10 and 12. Thus, these simulations reproduce in a newer WRF

105 version the convergence issue (or acoustic time-step dependency) raised by YF12 for the

106 DYCOMS-II RF02 case.

107   Vertical profiles of meteorological variables from the simulations using different

108 N_{aco} are depicted in Figure 3. Similar to the time-series results, the simulation using N_{aco} = 6

109 clearly diverges from the other simulations. Compared to the converged simulations, the

110 N_{aco} = 6 simulation has smoother vertical variations in potential temperature, water vapor

111 mixing ratio, and horizontal winds across the inversion, and smaller liquid water mixing

112 ratio and cloud fraction near the inversion. The profiles of turbulence characteristics, such

113 as variances and covariances, also have differences around the inversion and in the cloud

114 layer (not shown). Both the time series and vertical profiles show convergence of the cloud

115 variables in the simulations using $N_{aco} \geq 10$; thus, for the following discussion, we focus on

116 N_{aco} = 12 and 6, representing the converged and non-converged solutions, respectively.

117   To better understand the processes leading to non-convergence, one needs to

118 examine conditions when the non-converged and converged simulations start to diverge.

119 Figure 4 shows the vertical cross sections of perturbations of zonal wind, vertical velocity,

120 water vapor mixing ratio, and water vapor tendency due to condensation/evaporation 30

121 minutes into the simulation for N_{aco} = 6 and N_{aco} = 12. The perturbation is defined as the

122 deviation from the horizontal average; e.g., for zonal wind u, the perturbation $u' = u - \bar{u}$,

123 where $\bar{u}$ is the horizontal average. The N_{aco} = 6 simulation shows spurious motions near the

124 inversion for all four variables, which do not appear in the N_{aco} = 12 simulation. These

125 spurious motions are consistent with the smoother profiles in the N_{aco} = 6 simulation

126 (Figure 3). They reflect stronger mixing across the inversion to which the stratocumulus

127 clouds are very sensitive, as can be seen from the water vapor tendency due to

128 condensation/evaporation. It seems likely that the difference between the N_{aco} = 12 and

129   N_{aco} = 6 simulations in terms of cloud fraction, LWP, and cloud layer thickness, are caused

130   by these spurious motions. Interestingly, these spurious motions seem to grow slowly, over

131   minutes to tens of minutes compared to error growth that grows over several time steps

132   when violating CFL conditions, and the spurious motions saturate as the inversion gets

133   mixed out and becomes smoother during the integration (not shown).

134   Numerous sensitivity simulations are performed to identify conditions associated

135   with the convergence issue and the spurious motions near the inversion. The convergence

136   issue, as measured by the differences in cloud fraction and LWP between the N_{aco} = 12 and

137   N_{aco} = 6 simulations, and the magnitude of the spurious motions depend strongly on two

138   factors: horizontal wind speed and the moisture jump Δq_v across the inversion.

139   The impact of the first factor is shown by modifying the initial horizontal winds in

140   three sensitivity tests such that both the zonal and meridional winds are either (1) zero at

141   all levels, (2) constant in height but nonzero, i.e., with no wind shear, or (3) zero at the

142   inversion height but with a constant nonzero wind shear. In (2) the winds at the inversion

143   level in the control run are applied throughout the vertical domain while in (3) the wind

144   shear at the inversion level in the control run is applied throughout the depth of the model

145   domain. The modified wind profiles can be seen in Figure 1. The sensitivities to initial wind

146   conditions are shown in Figure 5. For the "no initial wind" and the "zero wind speed at

147   inversion" simulations, the results using N_{aco} = 6 and 12 are in close agreement. However,

148   for the "no wind shear" simulations, the difference between the two simulations is even

149   slightly larger than in the control set of simulations. Therefore, we conclude that wind

150   speed at the inversion height, but not wind shear, can cause the lack of convergence in the

151   DYCOMS-II RF02 case. Interestingly, although wind shear is generally considered to be an

152   important factor for cloud-top entrainment that contributes to cloud dissipation, shear is

153   not a controlling factor for the convergence issue.

154   To test the impact of the second factor, we modify the magnitude of the initial

155   moisture jump at the inversion height as shown in Figure 1. The DYCOMS-II RF02 case uses

156   a simple step function of total water mixing ratio to define the moisture jump at the

157   inversion that can be written as $\Delta q_v = q_{bot} - q_{top}$, where q_{top} is the water vapor mixing ratio

158   at the top of the inversion (5 g kg^{-1} for the control simulation) and q_{bot} is that at the bottom

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159   of the inversion (8.6 g kg^{-1}) resulting in Δq_v = 3.6 g kg^{-1} for the control. The magnitude of

160   the moisture jump in the sensitivity tests is reduced by 1 and 2 g kg^{-1} by setting q_{top} = 6 and

161   7 g kg^{-1}, respectively. The sensitivity of LWP to changes in the magnitude of the moisture

162   jump is shown in Figure 6. When the moisture jump is reduced by 1 g kg^{-1}, the divergence

163   in LWP between the N_{aco}=12 and N_{aco}=6 simulations is significantly reduced. When the

164   moisture jump is reduced by 2 g kg^{-1}, the two simulations appear to converge. Sensitivity

165   tests using even larger reductions in the moisture jump also show convergence for smaller

166   values of N_{aco} (not shown).

167   In both sets of sensitivity tests, when LWP in the N_{aco}=6 simulation converges to the

168   value in the corresponding N_{aco}=12 simulation, the spurious motions near the inversion

169   disappear. This further confirms that the spurious motions are responsible for the

170   divergence of simulations with different N_{aco}.

171   **2.2 ARM SGP stratocumulus case**

172   A second case with stratocumulus clouds over land serves to demonstrate the

173   generality of the issue, which has not been previously shown. On 13 May 2008, a very

174   strong temperature inversion and moisture jump occurred at the U.S. Department of

175   Energy (DOE) Atmospheric Radiation Measurement (ARM) Program Climate Research

176   Facility (CRF) at the Southern Great Plains (SGP) site (hereafter referred to as the ARM SGP

177   site) for the early-morning profiles of potential temperature, water vapor mixing ratio, and

178   horizontal winds at 1130 UTC (0530 LST) as shown in Figure 7. The water vapor mixing

179   ratio jumps from ~1 g kg^{-1} at ~1.6 km to ~13 g kg^{-1} at ~1.2 km. The profiles of potential

180   temperature and horizontal winds also show a very distinct two-layer structure below 5

181   km. This radiosonde profile has a much stronger moisture jump across the inversion and

182   stronger winds compared to the previous case (Figure 1), providing a very stringent test

183   for the convergence issue.

184   For this case, we use WRF version 3.6.1 with the Morrison double-moment

185   microphysics [Morrison et al. 2009] and RRTMG radiation schemes [Iacono et al. 2008]. It

186   will be shown later that the convergence issue and its solution do not depend on the choice

187   of physical parameterizations. Subgrid-scale (SGS) turbulent mixing is parameterized using

188   the 1.5-order TKE closure based on Deardorff [1980]. Surface latent and sensible heat

189   fluxes are prescribed from the ARM Best Estimates (ARMBE) dataset [ARM CRF, 1994; Xie

190   et al. 2010]. Surface roughness length is set to 0.04 m, surface albedo to 0.2, and emissivity

191   to 1.0 following Endo et al. [2015] based on their case from the Routine ARM Aerial Facility

192   (AAF) Clouds with Low Optical Water Depths (CLOWD) Optical Radiative Observations

193   (RACORO) campaign. No large-scale advective tendencies of temperature and moisture are

194   applied for this case. The simulations start at 1200 UTC and are integrated for 15 hours.

195   Random initial perturbations to the temperature profile shown in Figure 7 are applied in

196   the lowest 5 model layers. Model grid spacing is 100 m in the horizontal and ~20 m in the

197   vertical, and the model domain is 25.6 km by 25.6 km in the horizontal and 5 km in the

198   vertical with a 500 m deep sponge layer at the model top. We present four simulations that

199   differ only by their time steps (Δt=0.5 and 1.0 s) and acoustic sub-steps (N_{aco}=6 and 12).

200   Figure 8 shows the evolution of LWP and total cloud cover for the four simulations.

201   Total cloud cover from the Total Sky Imager (TSI; [ARM CRF, 2000]) and LWP from the

202   Microwave Radiometer Retrievals (MWRRET; [ARM CRF, 1996; Turner et al., 2007]) at the

203   ARM SGP site are shown for comparison. Some of the quantitative differences in terms of

204   cloud fraction are due to observed cirrus (seen in TSI images) at different times during the

205   simulation period, which are above the model top. Two primary discrepancies exist

206   between the simulations and reality. Unlike the observations, the model produces no cloud

207   during the first two simulated hours. And, the observed secondary peak in LWP and cloud

208   cover around 1600 LST is absent in the simulations. We believe these two discrepancies

209   are associated with either our simple initialization or the neglect of large-scale advective

210   forcing. Fortunately, for our present purposes the lack of clouds in the morning allows us to

211   examine the model behavior in clear-sky conditions, i.e., before the cloud microphysics

212   parameterization is active.

213   The convergence issue is produced in this case as well: the peak LWP value around

214   0930 LST drops from 97 g m^{-2} with Δt=0.5 s and N_{aco}=12, to 20 g m^{-2} with Δt=1.0 s and

215   N_{aco}=6, and peak cloud fraction drops from 1.0 to 0.8. As Figure 9 shows, the strength of the

216   spurious motions near the inversion also decreases with decreasing Δt and increasing N_{aco},

217   consistent with the increasing LWP. The spurious signal becomes negligible compared to

218   near-surface perturbations only when Δt=0.25 s and N_{aco}=12 (not shown). We found that
219   for this SGP test case, the same sensitivity exists as in the DYCOMS-II RF02 case regarding
220   the dependence of the spurious motions on the moisture jump and wind speed in the initial
221   condition (not shown).

3. The origin of the problem

223   To narrow down the necessary conditions for the existence of the spurious motions,
224   we perform a one-hour simulation of the ARM SGP stratocumulus case, with Δt=0.5 s and
225   N_{aco}=6, using a "stripped-down" 2-D version of WRF in which the microphysics, radiation,
226   and SGS turbulence parameterizations are all turned off. The Coriolis parameter is also set
227   to zero and the surface heat, moisture, and momentum fluxes are turned off. The model
228   resolution is kept the same as in the 3-D simulations discussed above. The model domain is
229   set to 5 km in the vertical direction and 4 km in the horizontal direction (i.e., 40 grid points
230   across). The initial wind in the horizontal direction is set to 12 m s^{-1} and uniform in the
231   vertical. Figure 10a shows the X-Z cross-section of perturbation vertical velocity, w', 40
232   minutes after initialization. The spurious motions near the inversion are quite similar to
233   those in the 3-D full-physics simulations in Section 2.2. This confirms that the spurious
234   motions   originate   from   the   dynamical   core,   rather   than   from   any   physical
235   parameterizations. We hereafter refer to this simulation as the "2-D test".

236   The analysis suggests that (1) the spurious motions near the inversion and the
237   sensitivity to time-stepping, especially acoustic sub-stepping, originate from the dynamical
238   core in WRF, and (2) the spurious motions are very sensitive to the moisture jump and
239   horizontal wind speed at the inversion layer. The following explanation of the WRF
240   dynamical core's treatment of moisture on the acoustic sub-steps elucidates how these
241   issues can arise.

242   Historically, the time-split numerics in WRF for integrating the dynamical equations
243   have   solved   prognostic   equations   for   the   flux-form   variables $\vec{V} = \mu_d \vec{v} = (U, V, \Omega)$, $\Theta =$
244   $\mu_d \theta$, and μ_d on the acoustic sub-steps since these equations contain the terms that are
245   fundamentally responsible for the propagation of acoustic and gravity waves [see Klemp et
246   al. 2007; Skamarock et al. 2008 and references therein]. Here, $\vec{v}$ represents the covariant

247   components of the velocity, θ is the potential temperature, and μ_d is the dry air mass per

248   unit area in the column (i.e., the dry hydrostatic pressure difference between the surface

249   and the model top). Moisture variables (including q_v) and other scalars are advanced on the

250   larger dynamic time steps (i.e., the Runge-Kutta time steps). The pressure, p, must also be

251   solved on the acoustic sub-steps, and is obtained from the ideal gas law:

252
$$p = p_0 \left(\frac{R_d \Theta \left(1 + \frac{R_v}{R_d} q_v^t \right)}{p_0 \alpha_d \mu_d} \right)^\gamma \qquad (1)$$

253   where p_0 is a reference pressure, R_d and R_v are the gas constants of dry air and water vapor,

254   respectively, and $\gamma = c_p/c_v$ is the ratio of heat capacities for dry air. $\alpha_d = 1/\rho_d$, is the

255   inverse of dry air density. Recognizing that the moisture variables are stepped forward on

256   the dynamic time steps and therefore are not varying during the acoustic sub-steps, q_v is

257   held fixed at q_v^t during the acoustic sub-steps, where the t superscript indicates values at

258   the current Runge-Kutta time step.

259   This approach in time-splitting the numerical integration has proven to be robust

260   over a wide spectrum of applications. However, in simulations with a large vertical

261   gradient in moisture (near discontinuity) at high resolution, ignoring variations in q_v

262   during the acoustic sub-steps may no longer serve as an accurate approximation.

263   **4. The solution**

264   To resolve this deficiency, we define a moist potential temperature $\Theta_m = \mu_d \theta_m =$

265     $\mu_d \theta \left(1 + \frac{R_v}{R_d} \right) q_v$ as the prognostic variable instead of Θ. With this modification, the

266   prognostic equation for Θ is replaced by:

267
$$\partial_t \Theta_m + \left(\nabla \cdot \vec{V} \theta_m \right)_\eta = F_{\Theta_m}, \qquad (2)$$

268   which has exactly the same form as the Θ equation (see Klemp et al. 2007, Equation (24)),

269   with θ_m replacing θ and F_Θ suitably modified to F_{Θ_m}. The gas law is then expressed as

270
$$p = p_0 \left(\frac{R_d \Theta_m}{p_0 \alpha_d \mu_d} \right)^\gamma, \qquad (3)$$

271 in which q_v no longer appears explicitly. Thus, by folding q_v into the Θ_m variable, variations

272 in q_v are implicitly accommodated during the acoustic sub-steps without the need to update

273 q_v explicitly on these steps. The pressure can then be advanced on these sub-steps using

274 Equation (3), in which all variables on the right-hand side are also updated on these same

275 sub-steps. This procedure for representing moisture on the acoustic sub-steps using a

276 prognostic Θ_m variable is the same as that implemented by Skamarock et al. [2012] for the

277 time-split numerical scheme used by the global Model for Prediction Across Scales (MPAS).

278   The effectiveness of the proposed modification is demonstrated in the next two

279 sections for the DYCOMS-II RF02 and ARM SGP stratocumulus cases.

4.1 DYCOMS-II RF02 case

281   The dynamical core modifications in WRF eliminate the spurious fluctuations for the

282 DYCOMS-II RF02 case (not shown) and the convergence problem is also resolved for this

283 case, as shown in Figure 11. While the original model produces different values of LWP,

284 cloud-top and base heights, and total cloud fraction for N_{aco}=6 and N_{aco}=12, the simulations

285 using the modified model show a clear convergence.

4.2 ARM SGP stratocumulus case

287   Repeating the 2-D test from Section 3 with the modified dynamical core shows

288 clearly that the spurious motions near the inversion are absent (Figure 10b). We also

289 repeat the four 3-D full physics simulations described in Section 2.2 using WRF v3.6.1 with

290 the modified dynamical core. Figure 12 shows the evolution of total cloud fraction and LWP

291 for the four simulations in the same format as Figure 8. Comparing the two figures, one can

292 clearly see that the convergence issue is resolved with the modified model. The simulated

293 total cloud fraction and LWP all converge to virtually the same values. Note also that the

294 original model using Δt=0.5 s and N_{aco}=12 (the black solid lines in Figure 8) produces total

295 cloud fraction and LWP very close to those produced by the modified model. This implies

296 that for this particular case and setup, Δt=0.5 s and N_{aco}=12 are close to the lowest

297 temporal resolution needed for convergence without the modifications and, with the

298 modifications one can expect the simulations to converge at Δt=1.0 s and N_{aco}=6 or even

299 less conservative time settings. The spurious motions near the inversion (Figure 9) are also
300 completely eliminated in the simulations with the modified model (not shown).

301 **5. Conclusions and discussion**

302   In this study, we investigate the sensitivity of cloud properties to time-stepping
303 choices in WRF large-eddy simulations of stratocumulus clouds, which has previously been
304 noted by Yamaguchi and Feingold [2012]. We examine the sensitivity in two stratocumulus
305 cases: one marine, the GCSS DYCOMS-II RF02 case, and one continental, a stratocumulus
306 case over the ARM SGP site on 13 May 2008.

307   We demonstrate that, in both cases, the sensitivity is associated with spurious
308 motions near the inversion at the boundary layer top. Decreasing the temporal resolution,
309 by increasing Δt or decreasing N_{aco}, increases the magnitude of the spurious signal, giving
310 rise to the sensitivity seen in the cloud fraction and LWP. The spurious signal is very
311 sensitive to (1) the moisture jump across the inversion (the stronger the moisture jump,
312 the stronger the signal) and (2) the mean horizontal wind in the inversion layer (the
313 spurious motions vanish if we set the mean wind to zero). Furthermore, the spurious
314 motions persist in a 2-D setup of WRF with all physical parameterizations and external
315 forcings turned off (including mean wind shear and Coriolis effect) for the ARM SGP
316 stratocumulus case. All these clues point to the WRF dynamical core being the issue, rather
317 than the physics parameterizations, and examination of assumptions within the dynamical
318 core suggests the problem lies in the treatment of moisture on the acoustic sub-steps.

319   In WRF, water vapor mixing ratio (q_v) is integrated only on the larger dynamic time
320 steps. In the calculation of pressure on the acoustic sub-steps, the effect of moisture
321 perturbations from sub-step to sub-step is assumed to be negligible. This assumption
322 becomes problematic in simulations with very large vertical moisture gradients, where
323 moisture perturbations during the acoustic sub-steps can become large.

324   We suggest modifying the WRF dynamical core such that Θ_m is the prognostic
325 variable instead of Θ. This equates to using Equation (3) (instead of (1)) in the calculation
326 of pressure on the acoustic sub-steps. Our results show that this modification prevents the
327 spurious motions from forming and removes the sensitivity to time-stepping choices in

328 both the marine and continental stratocumulus cases. WRF version 3.7, now available to
329 the community, has incorporated our modification as an option for LES.

330   Implied in the results presented here is that the modification to the dynamical core
331 permits the use of longer time steps and fewer acoustic sub-steps than in the original
332 version of WRF. This has the advantage of making the model simulations computationally
333 cheaper. The purpose of this paper is not to test the limits of the time step settings;
334 however, we note that time steps twice as long as previously used now give similar results
335 as shorter time steps.

336   In retrospect, the convergence issue arises through the choice of prognostic
337 variables used in the dynamic equations under the time-split integration framework. The
338 two stratocumulus cases discussed here highlight the scenario where water vapor plays an
339 active role in the "dry" dynamics, hence the need to carry the moist potential temperature
340 θ_m as a prognostic variable on the acoustic time step.

341   The problems that we have illustrated for LES applications within the original WRF
342 model formulation (using θ as opposed to θ_m as a prognostic variable) have not been
343 observed in other LES models (e.g. the LES models described in Stevens et al. 2005 and
344 references therein). Most existing moist LES models use as a prognostic variable some form
345 of potential temperature coupled with moisture, for example the virtual potential
346 temperature or some form of moist static energy, and as such, moisture is taken into
347 account in the dynamic time step. In addition, not all LES models are compressible, and of
348 those that admit acoustic waves only a few use a time-split integration technique that
349 would be susceptible to the problems we have discussed. Recently developed
350 nonhydrostatic compressible atmospheric solvers using height coordinates and cast in
351 conservative form are using θ_m or θ_v as illustrated by MPAS [Skamarock et al. 2012] and
352 the ICOsahedral Nonhydrostatic (ICON) model [Zängl et al. 2014], respectively. Models
353 using non-conservative formulations, such as the Coupled Ocean-Atmosphere Mesoscale
354 Prediction System (COAMPS, Hodur 1997), the Advanced Regional Prediction System
355 (ARPS, Xue et al. 2000), and the Regional Atmospheric Modeling System (RAMS, Pielke et
356 al. 1992) have not incorporated moisture into the thermodynamic equation on the acoustic

357   steps in their time-split integrations. A number of new nonhydrostatic compressible model

358   formulations are being developed because of the capabilities of new computers to

359   accommodate global cloud-permitting simulations, some of which are time-split. We do

360   not know if the problems exhibited in the LES regime exist in some form at coarser grid

361   spacings, and existing and new nonhydrostatic solver formulations should be examined for

362   susceptibility to these problems. In the future, the development of compressible dynamical

363   cores using similar time-split methods should take into account our finding in the design of

364   the equation system, especially when choosing prognostic variables for the system.

365

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517 **Figures**
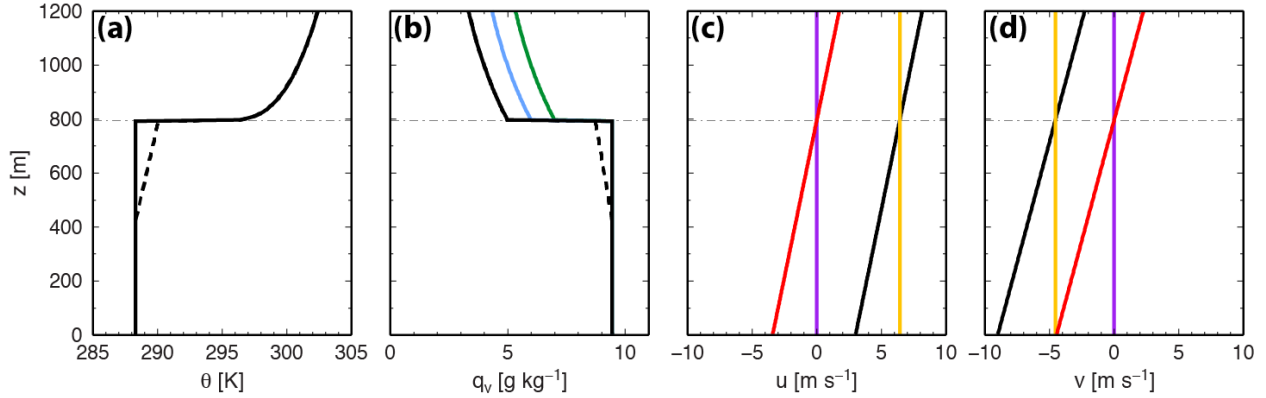
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519 Figure 1: Initial profiles of (a) potential temperature (b) water vapor mixing ratio, and (c)
520 zonal and (d) meridional winds for the DYCOMS-II RF02 case control simulations (black
521 lines) and sensitivity simulations with a reduced moisture jump (1 g kg^{-1} in blue line; 2 g
522 kg^{-1} in green) and modified winds ('no wind' run with purple lines; 'no shear' run with
523 yellow lines; 'zero speed at inversion' run with red lines). The black dashed lines indicate
524 the values after condensation occurs in the first time step of the control simulation.
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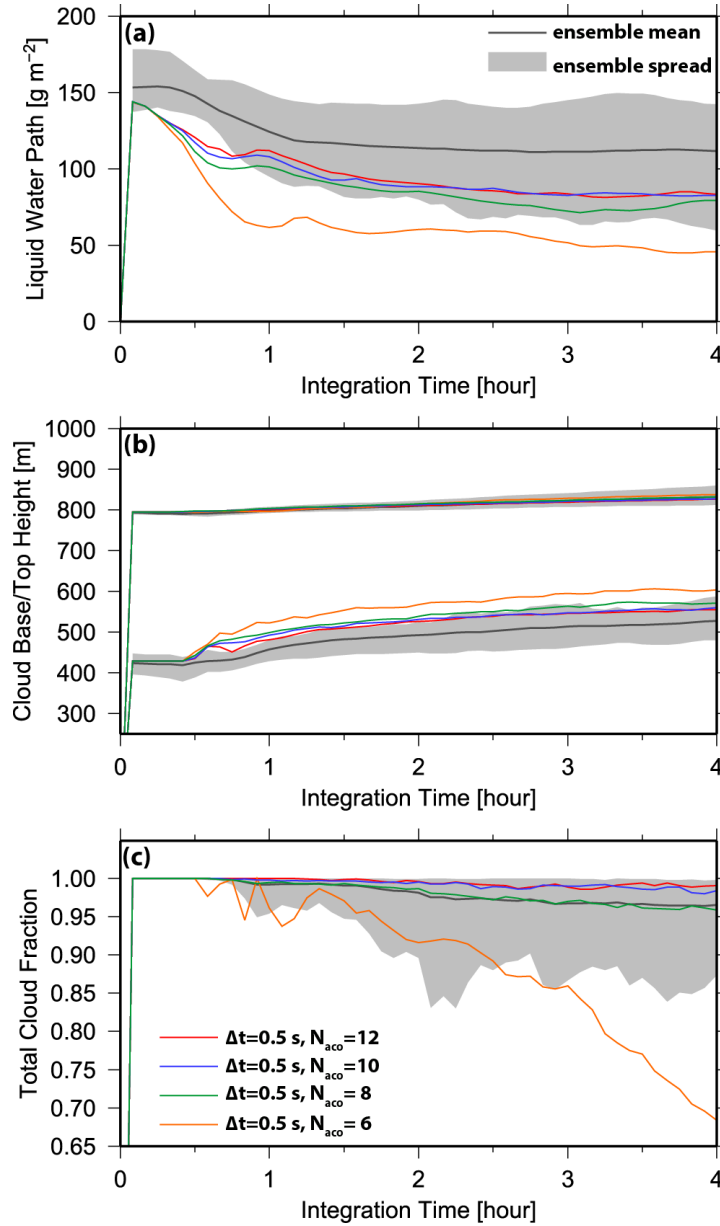
Figure 2: Time series of (a) LWP, (b) cloud-top and cloud-base heights, and (c) cloud fraction from the DYCOMS-II RF02 simulations using Δt = 0.5 s and N_{aco} = 6, 8, 10, and 12 (see colors in legend). Following Ackermann et al. [2009], cloud-top is defined as the height where domain-averaged total water = 8 g kg^{-1} and cloud fraction is defined as domain fraction of columns with column LWP > 20 g m^{-2}. The ensemble mean (black line) and spread (gray shading) from simulations in the inter-comparison described in Ackerman et al. [2009] are shown for comparison.
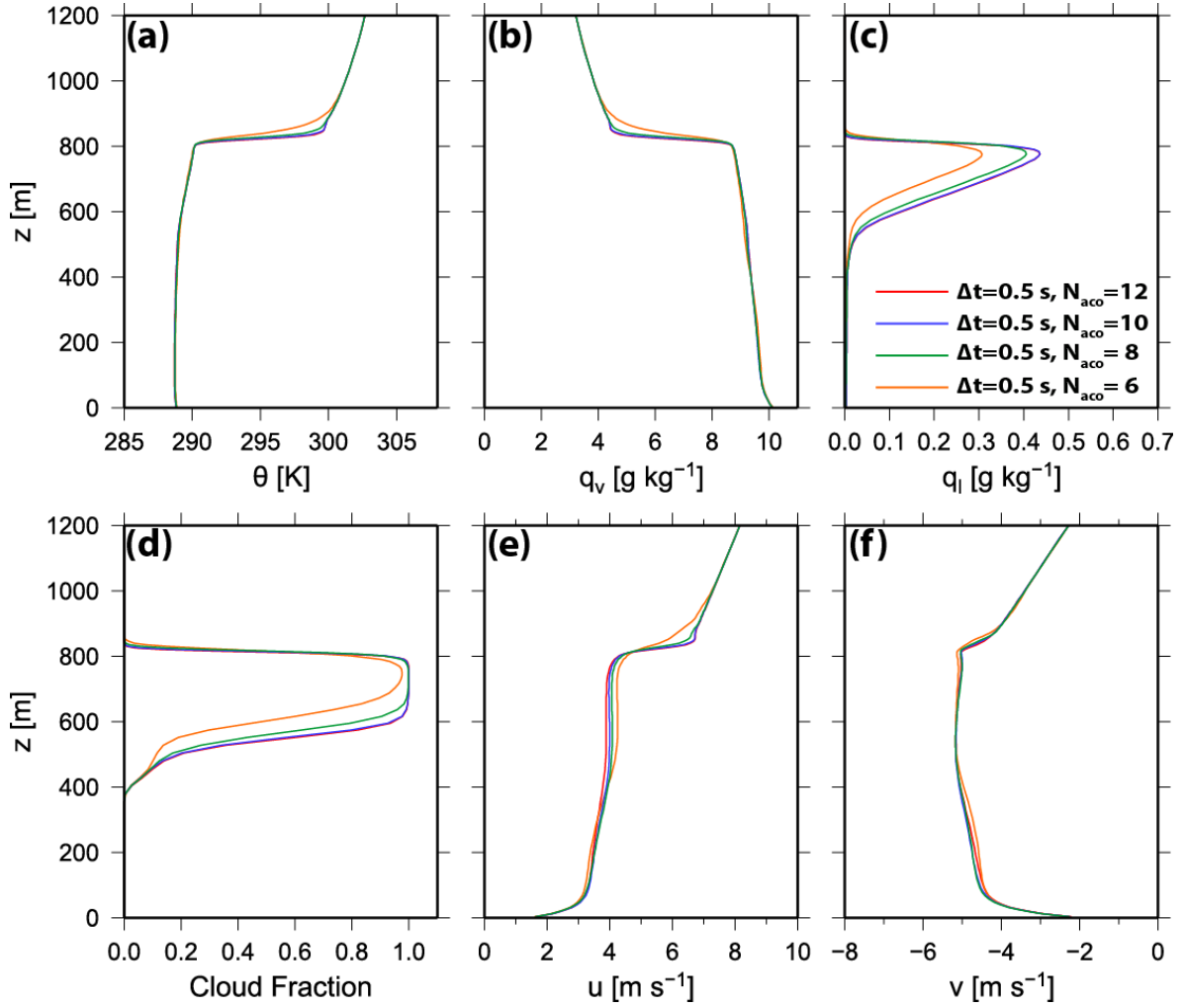
Figure 3: Vertical profiles of (a) potential temperature, (b) water vapor mixing ratio, (c) liquid water mixing ratio, (d) cloud fraction, and (e) zonal and (f) meridonal winds. Values are averages for hours 2 to 4 of the integration period for DYCOMS-II RF02 simulations using N_{aco} = 6, 8, 10, and 12.
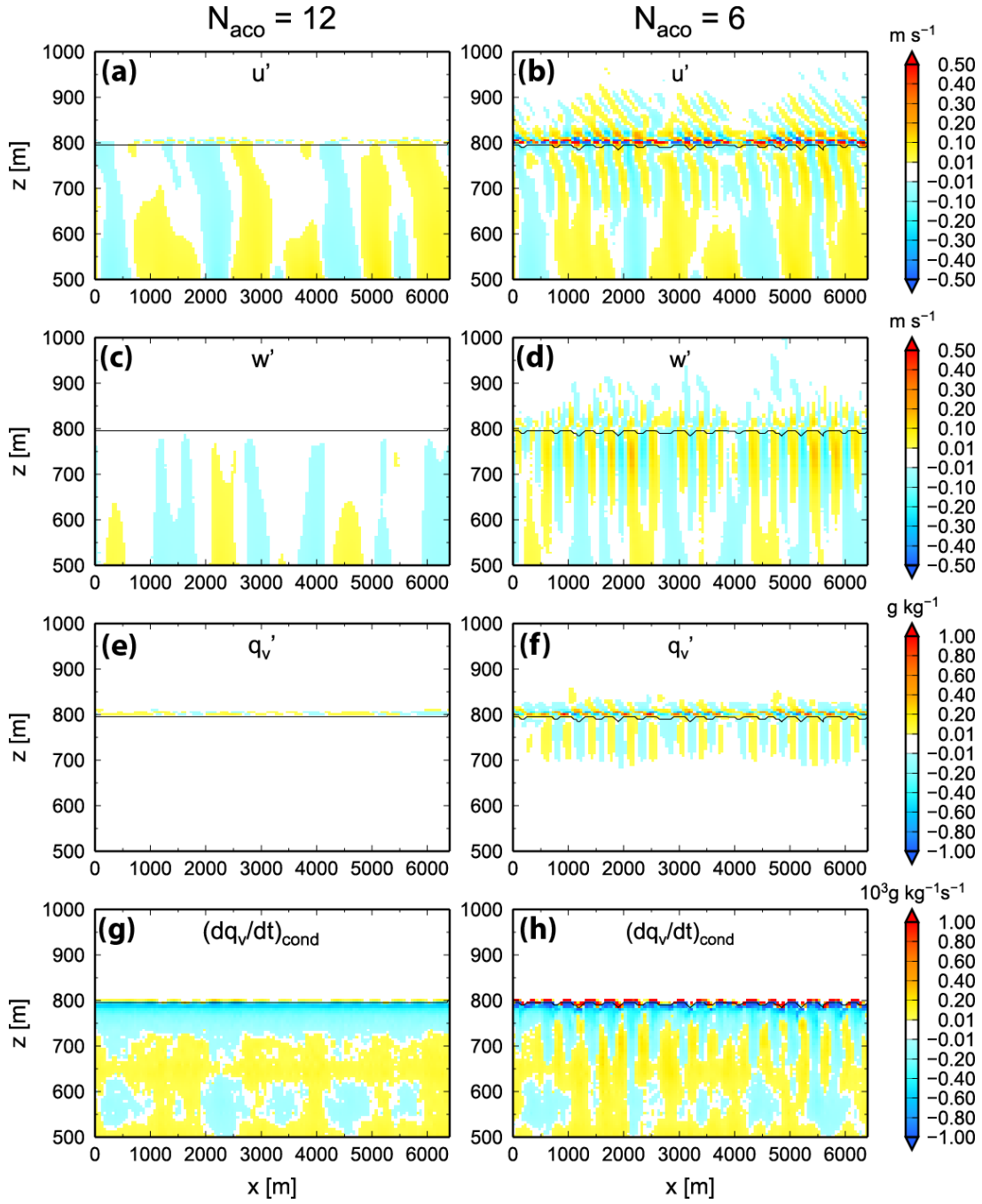
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542 Figure 4: Vertical cross section of (a-b) zonal wind perturbation, u' (m s^{-1}), (c-d) vertical

543 wind perturbation, w' (m s^{-1}), (e-f) water vapor mixing ratio perturbation, q_v' (g kg^{-1}). and

544 (g-h) water vapor tendency due to evaporation and condensation, $(dq_v/dt)_{cond}$ (g kg^{-1} s^{-1}),

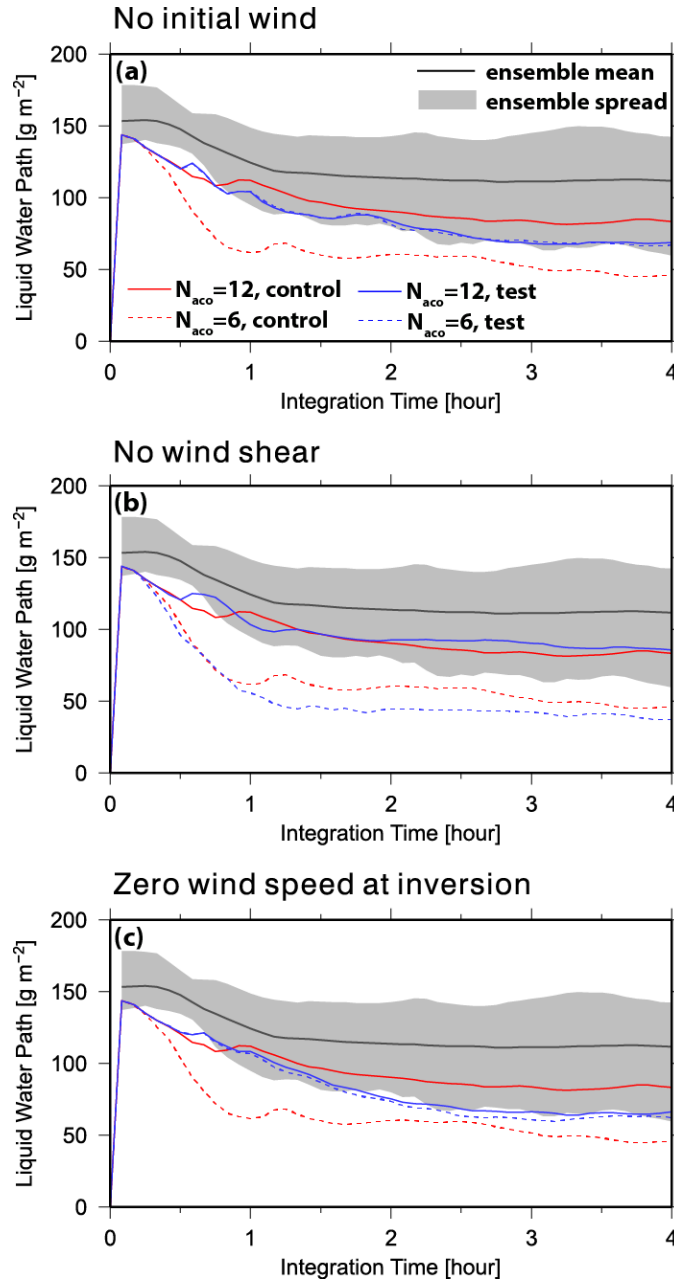545 30 minutes into the DYCOMS-II RF02 simulation period using N_{aco} = 6 (right) and 12 (left).

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547

Figure 5: Time series of LWP (g m^{-2}) from the initial wind sensitivity tests of the DYCOMS-II RF02 case, as described in the text. Large differences between the solid and dashed blue lines indicate the presence of the convergence issue. Note the sensitivity simulations converge to a different solution than the control simulation (solid red line) because of changes in initial wind profiles.
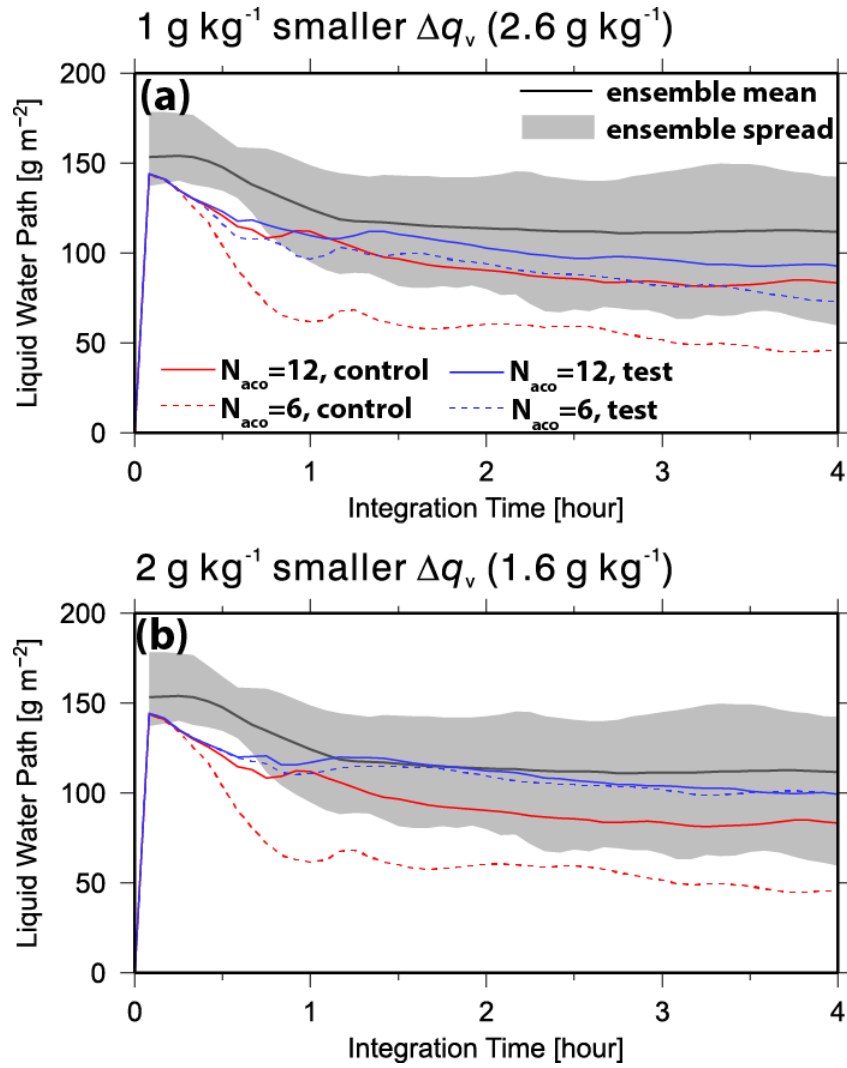
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1 g kg^{-1} smaller Δq_v (2.6 g kg^{-1})

(Figure panel (a): Liquid Water Path [g m^{-2}] vs Integration Time [hour], with legend: ensemble mean, ensemble spread, N$_{aco}$=12, control; N$_{aco}$=6, control; N$_{aco}$=12, test; N$_{aco}$=6, test)

2 g kg^{-1} smaller Δq_v (1.6 g kg^{-1})

(Figure panel (b): Liquid Water Path [g m^{-2}] vs Integration Time [hour])

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555   Figure 6: As in Figure 5, except for sensitivity tests with reduced moisture jumps across the

556   inversion.

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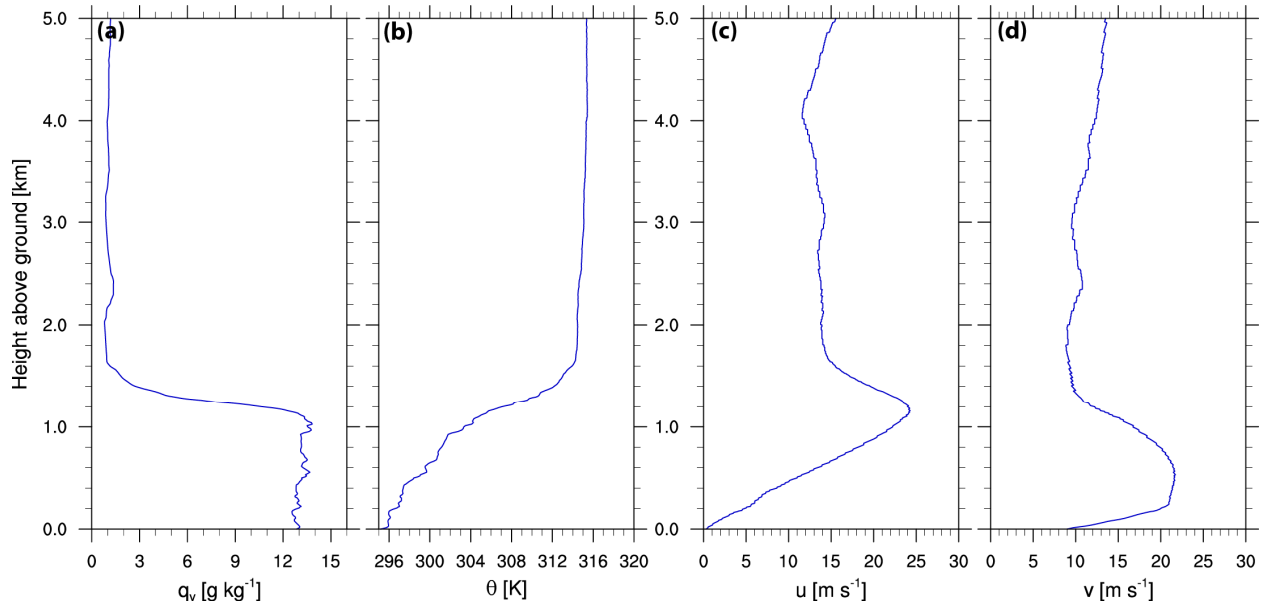
559   Figure 7: The profiles of (a) water vapor mixing ratio (g kg⁻¹), (b) potential temperature

560   (K), (c) zonal and (d) meridional winds (m s⁻¹) from the 1130 UTC sounding at the ARM

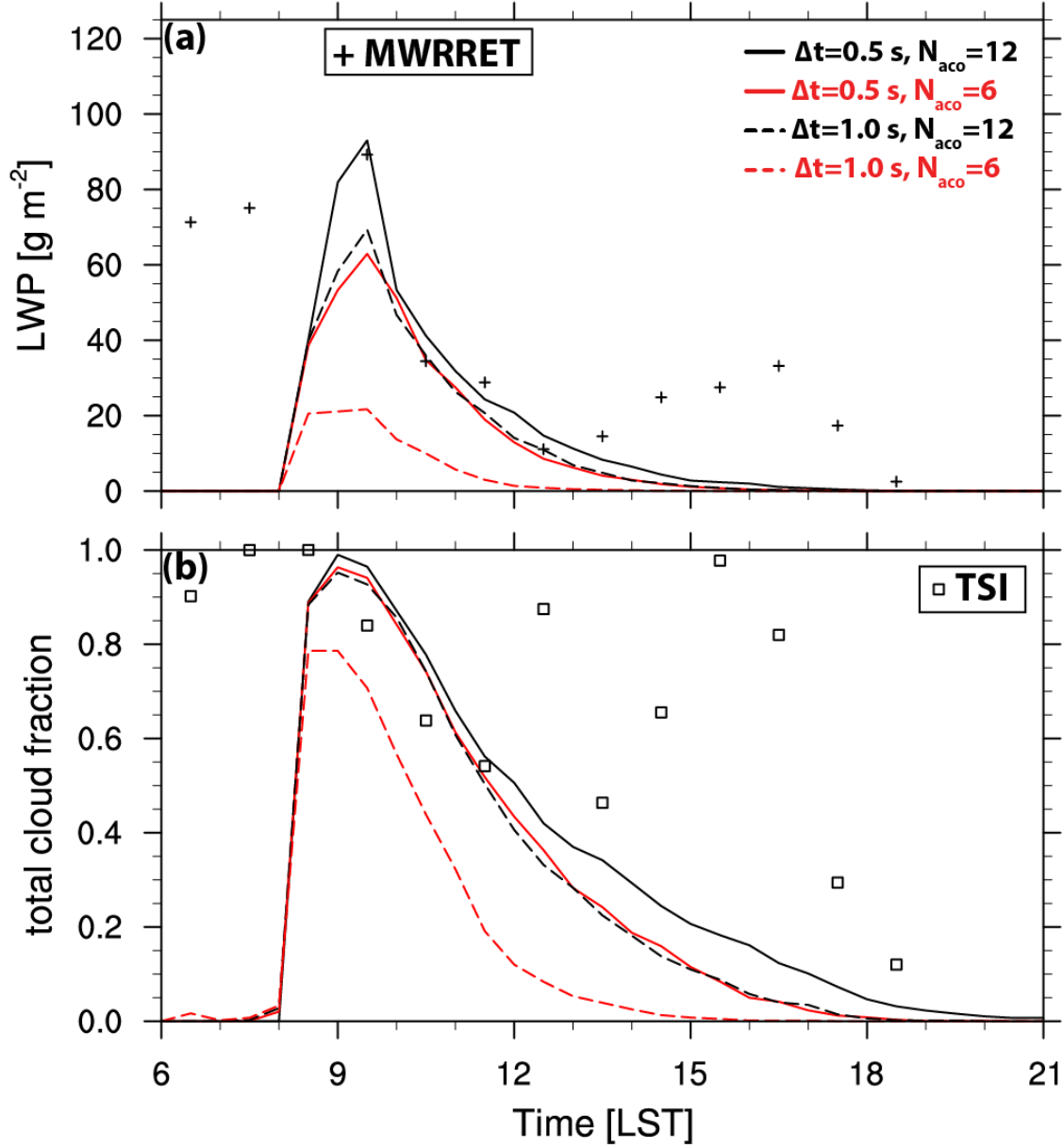561   SGP site on May 13 2008.

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Figure 8: (a) The LWP (g m^{-2}) and (b) total cloud fraction of the four WRF simulations of
the SGP stratocumulus case with different time stepping choices. MWRRET LWP and TSI
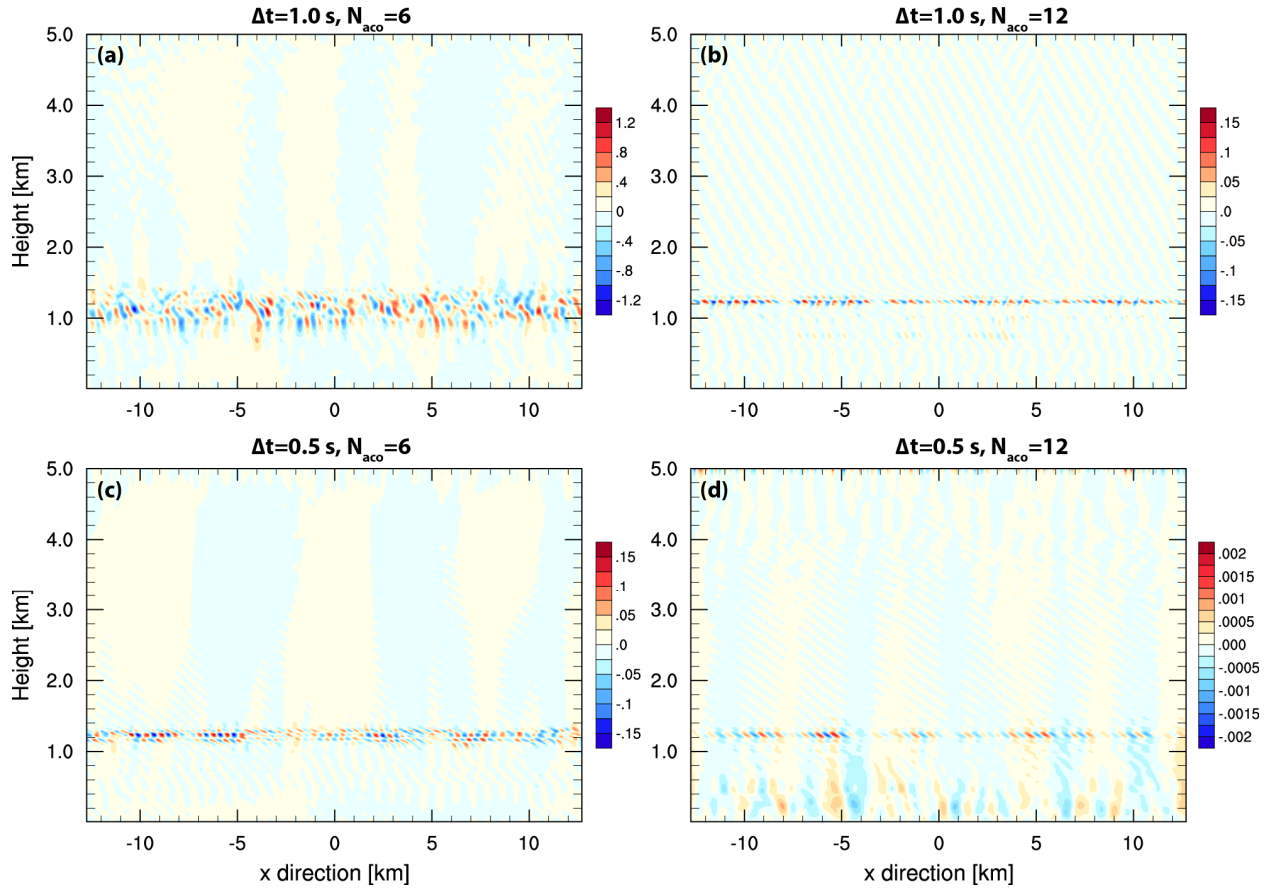cloud fraction observations from the ARM SGP site are shown for comparison.

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Figure 9: The X-Z cross-section of w' (deviation from horizontal mean, m s^{-1}) at t=30 minutes for the WRF simulations of the SGP stratocumulus case using (a) Δt=1.0 s, N_{aco}=6, (b) Δt=1.0 s, N_{aco}=12, (c) Δt=0.5 s, N_{aco}=6 and (d) Δt=0.5 s, N_{aco}=12. Note that different panels use different color scales.
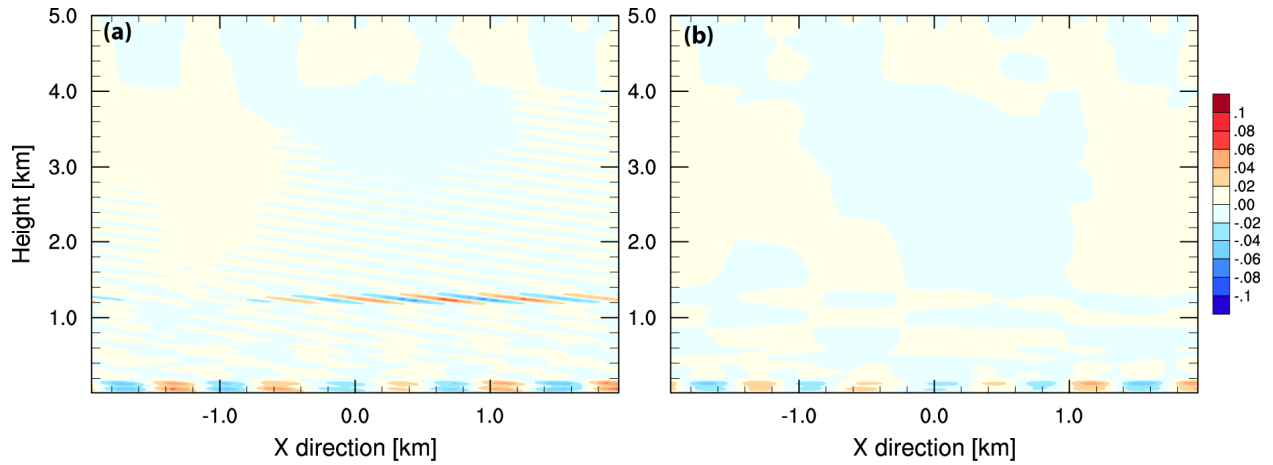
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Figure 10: The X-Z cross-section of w' (deviation from horizontal mean, m s^{-1}) at t=40 minutes for the WRF 2-D test simulation of the SGP stratocumulus case with (a) the original WRF and (b) the modified dynamical core in WRF.
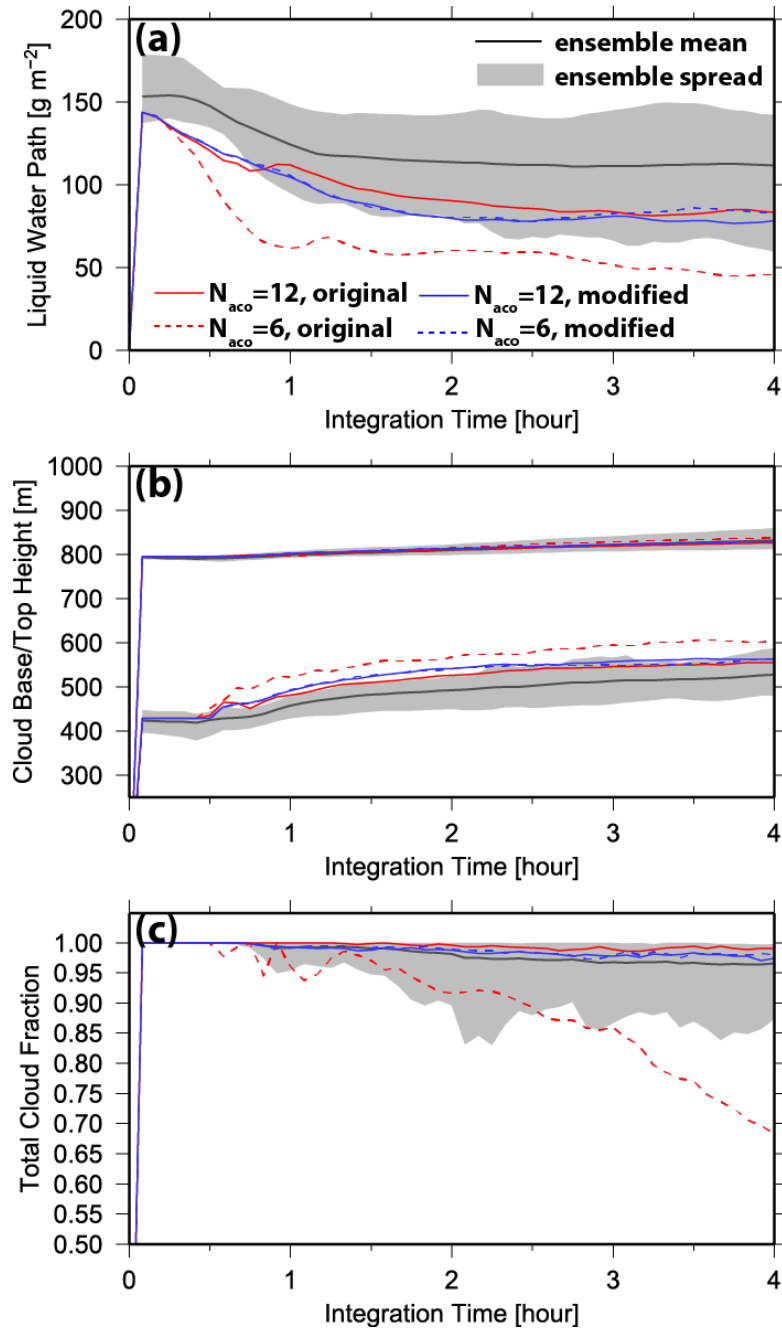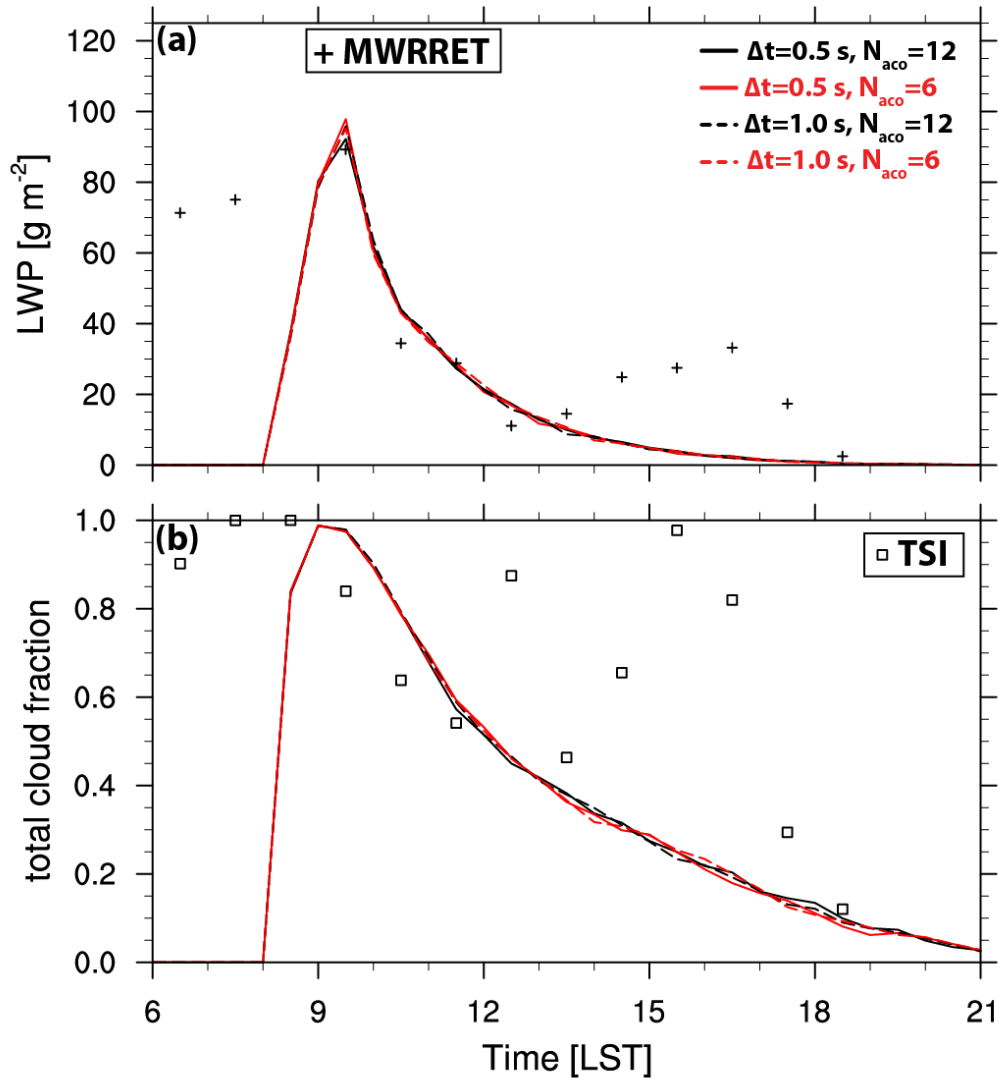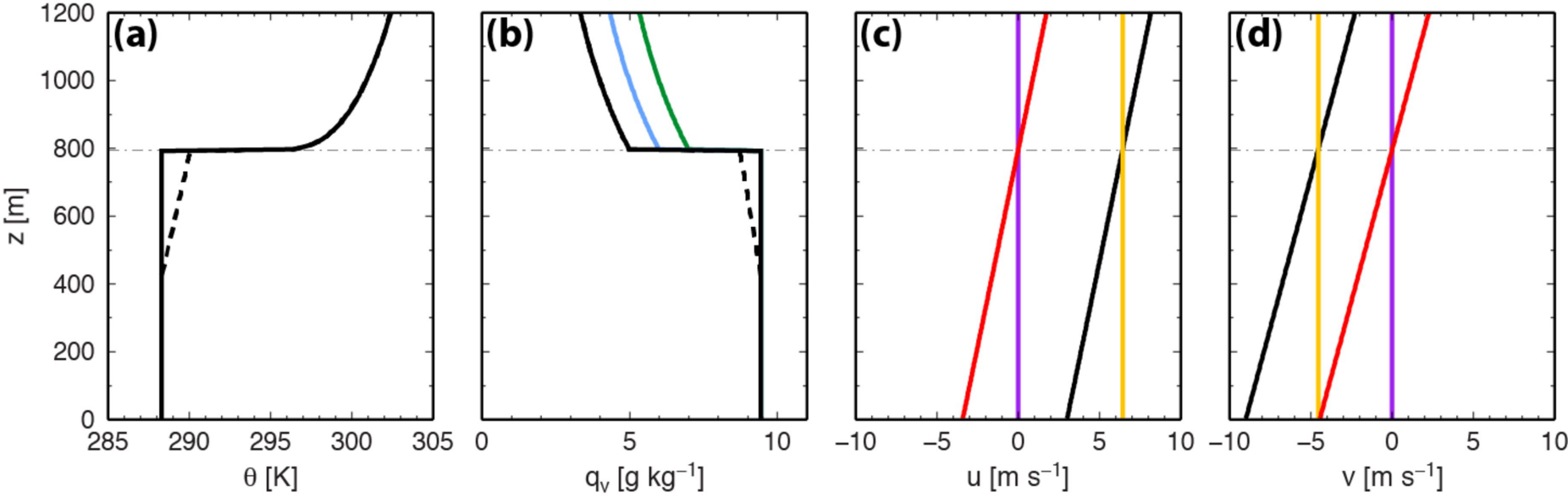
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580 Figure 11: As in Figure 2, except for the simulations using the modified dynamical core for

581 the DYCOMS-II RF02 case.

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584 Figure 12: (a) The LWP (g m^{-2}) and (b) total cloud fraction of the four WRF simulations

585 with modified dynamical core of the SGP stratocumulus case with different time stepping

586 choices. MWRRET LWP and TSI cloud fraction observations from the ARM SGP site are

587 shown for comparison.

Figure legend (panel c):
- $\Delta t = 0.5$ s, $N_{aco} = 12$
- $\Delta t = 0.5$ s, $N_{aco} = 10$
- $\Delta t = 0.5$ s, $N_{aco} = 8$
- $\Delta t = 0.5$ s, $N_{aco} = 6$

Panel (a): x-axis θ [K], y-axis z [m]
Panel (b): x-axis q_v [g kg^{-1}], y-axis z [m]
Panel (c): x-axis q_l [g kg^{-1}], y-axis z [m]
Panel (d): x-axis Cloud Fraction, y-axis z [m]
Panel (e): x-axis u [m s^{-1}], y-axis z [m]
Panel (f): x-axis v [m s^{-1}], y-axis z [m]

$N_{aco} = 12$ $N_{aco} = 6$

(a) u' (b) u' m s^{-1}

(c) w' (d) w' m s^{-1}

(e) q_v' (f) q_v' g kg^{-1}

(g) $(dq_v/dt)_{cond}$ (h) $(dq_v/dt)_{cond}$ 10^3g kg^{-1}s^{-1}

No initial wind

(a)

Liquid Water Path [g m^{-2}]

— ensemble mean
▒ ensemble spread

— $N_{aco}=12$, control   — $N_{aco}=12$, test
- - - $N_{aco}=6$, control   - - - $N_{aco}=6$, test

Integration Time [hour]

No wind shear

(b)

Liquid Water Path [g m^{-2}]

Integration Time [hour]

Zero wind speed at inversion

(c)

Liquid Water Path [g m^{-2}]

Integration Time [hour]

(a) 1 g kg⁻¹ smaller Δq_v (2.6 g kg⁻¹)

- ensemble mean
- ensemble spread
- N_{aco}=12, control
- N_{aco}=6, control
- N_{aco}=12, test
- N_{aco}=6, test

(b) 2 g kg⁻¹ smaller Δq_v (1.6 g kg⁻¹)

Liquid Water Path [g m⁻²] vs Integration Time [hour]

Δt=1.0 s, N_aco=6 — (a)

Δt=1.0 s, N_aco=12 — (b)

Δt=0.5 s, N_aco=6 — (c)

Δt=0.5 s, N_aco=12 — (d)

(a) Liquid Water Path [g m^{-2}] vs Integration Time [hour]

- ensemble mean
- ensemble spread
- N_{aco}=12, original
- N_{aco}=6, original
- N_{aco}=12, modified
- N_{aco}=6, modified

(b) Cloud Base/Top Height [m] vs Integration Time [hour]

(c) Total Cloud Fraction vs Integration Time [hour]

(a) $+$ MWRRET

Δt=0.5 s, N_{aco}=12
Δt=0.5 s, N_{aco}=6
Δt=1.0 s, N_{aco}=12
Δt=1.0 s, N_{aco}=6

LWP [g m^{-2}]

120
100
80
60
40
20
0

(b)

$\square$ TSI

total cloud fraction

1.0
0.8
0.6
0.4
0.2
0.0

Time [LST]

6   9   12   15   18   21